A Fast Algorithm for Multilevel Thresholding

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Otsu reference proposed a criterion for maximizing the between-class variance of pixel intensity to perform picture thresholding. However, Otsu's method for image segmentation is very time-consuming because of the inefficient formulation of the between-class variance. In this paper, a faster version of Otsu's method is proposed for improving the efficiency of computation for the optimal thresholds of an image. First, a criterion for maximizing a modified between-class variance that is equivalent to the criterion of maximizing the usual between-class variance is proposed for image segmentation. Next, in accordance with the new criterion, a recursive algorithm is designed to efficiently find the optimal threshold. This procedure yields the same set of thresholds as the original method. In addition, the modified between-class variance can be pre-computed and stored in a look-up table. Our analysis of the new criterion clearly shows that it takes less computation to compute both the cumulative probability (zeroth order moment) and the mean (first order moment) of a class, and that determining the modified between-class variance by accessing a look-up table is quicker than that by performing mathematical arithmetic operations. For example, the experimental results of a five-level threshold selection show that our proposed method can reduce down the processing time from more than one hour by the conventional Otsu's method to less than 107 seconds.

Keywords: Otsu's thresholding, image segmentation, picture thresholding, multilevel thresholding, recursive algorithm

1. INTRODUCTION

Thresholding is an important technique for image segmentation that tries to identify and extract a target from its background on the basis of the distribution of gray levels or texture in image objects. Most thresholding techniques are based on the statistics of the one-dimensional (1D) histogram of gray levels and on the two-dimensional (2D) co-occurrence matrix of an image. Many 1D thresholding methods have been investigated [1-9]. Locating the thresholds can be proceed in parametric or nonparametric approaches [1, 4, 13]. In parametric approaches, the gray level distribution of an object class leads to finding the thresholds. For instance, in Wang and Haralick's study [5],

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the pixels of an image are first classified as either edge non-edge pixels. According to their local neighborhoods, edge pixels are then classified as being relatively dark or relatively bright. Next, one histogram is obtained for those edge pixels which are dark and another for those edge pixels which are bright. The highest peaks of these two histograms are chosen as the thresholds. Moment preserving thresholding is a parametric method which segments the image based on the condition that the thresholded image has the same moments as the original image [3]. In nonparametric approaches, the thresholds are obtained in an optimal manner according to some criteria. For instance, Otsu's method chooses the optimal thresholds by maximizing the between-class variance with an exhaustive search [2]. In Pun's method [7], as modified by Kapur et al. [8], the picture threshold is found by maximizing the entropy of the histogram of gray levels of the resulting classes. Other, some 1D thresholding techniques extend from bi-level threshold selection to multilevel threshold selection [1-5]. In contrast to 1D thresholding methods, 2D methods essentially do image segmentation by using spatial information in an image [10-15]. Kirby and Rosenfeld proposed a 2D thresholding method that simultaneously considers both the pixel gray level and the local statistics of its neighboring pixels [12]. One particular 2D method is entropic thresholding, which makes use of spatial entropy to find the optimal thresholds. Abutaleb [13], and Pal and Pal [14] proposed that optimal thresholds can be selected by maximizing the sum of the posterior entropies of two classes. However, their method is very time-consuming at determining the 2D total entropy of the resulting two classes. As a result, Chen et al. proposed a two-stage approach to search for the optimal threshold of 2D entropic thresholding so that the computation complexity can be reduced to $O(L^{8/3})$ for an image with L gray levels [10]. Recently, Gong et al. [11] designed a recursive algorithm for 2D entropic thresholding to further reduce the computation complexity to $O(L^2)$. However, it is still inefficient to apply this algorithm to 1D multilevel thresholding selection, owing to their computation of threshold without taking advantage of the recursive structure of entropy measures.

The Sahoo et al. study on global thresholding, [6] concluded that Otsu's method was one of the better threshold selection methods for general real world images with regard to uniformity and shape measures. However, Otsu's method uses an exhaustive search to evaluate the criterion for maximizing the between-class variance. As the number in classes of an image increases, Otsu's method takes too much time to be practical for multilevel threshold selection. To determine the 1D threshold of an image efficiently, we propose a modified between-class variance for Otsu's method. The recursive form of the proposed modified between-class variance will considerably decrease the computation of summing both the zeroth order moment and the first order moment of the probability density function up to the *k*th gray level that will be described in Section 4. Then, a look-up table is derived from the recursive form of the modified between-class variance. It is designed to replace the computation of the modified between-class variance for the 1D multilevel thresholding. Compared to the conventional Otsu's method with a recursive form of the between-class variance, our proposed algorithm can increase the speed of computation by 22 times based on experimental results.

In Section 2, the Otsu's method for image thresholding is briefly reviewed. In Section 3, a modification of Otsu's thresholding method for 1D multilevel threshold selection is discussed. In section 4, a fast 1D multilevel thresholding algorithm based on the recursive forms of ω_k and $\mu(k)$ and the look-up table of the modified between-class variance is described in detail. Section 5 shows the experimental results. Finally, Section 6 gives a brief conclusion.

2. OTSU'S METHOD FOR IMAGE THESHOLDING

An image is a 2D grayscale intensity function, and contains N pixels with gray levels from 1 to L. The number of pixels with gray level i is denoted f_i, giving a probability of gray level i in an image of

$$p_i = f_i / N. \tag{1}$$

In the case of bi-level thresholding of an image, the pixels are divided into two classes, C_1 with gray levels [1, …, t] and C_2 with gray levels [t+1, …, L]. Then, the gray level probability distributions for the two classes are

C_1: $p_1/\omega_1(t), \ldots. p_t/\omega_1(t)$ and

C_2: $p_{t+1}/\omega_2(t), p_{t+2}/\omega_2(t), \ldots, p_L/\omega_2(t)$,

$$\text{where } \omega_1(t) = \sum_{i=1}^{t} p_i \tag{2}$$

and

$$\omega_2(t) = \sum_{i=t+1}^{L} p_i. \tag{3}$$

Also, the means for classes C_1 and C_2 are

$$\mu_1 = \sum_{i=1}^{t} i\, p_i / \omega_1(t) \tag{4}$$

and

$$\mu_2 = \sum_{i=t+1}^{L} i\, p_i / \omega_2(t). \tag{5}$$

Let μ_T be the mean intensity for the whole image. It is easy to show that

$$\omega_1\mu_1 + \omega_2\mu_2 = \mu_T \tag{6}$$

$$\omega_1 + \omega_2 = 1 \tag{7}$$

Using discriminant analysis, Otsu defined the between-class variance of the thresholded image as [2]

$$\sigma_B^{\ 2} = \omega_1 (\mu_1 - \mu_T)^2 + \omega_2 (\mu_2 - \mu_T)^2. \tag{8}$$

For bi-level thresholding, Otsu verified that the optimal threshold t* is chosen so that the between-class variance σ_B^2 is maximized; that is,

$$t^* = \underset{1 \le t < L}{\text{Arg} \quad \text{Max}} \ \{\sigma_B^{\ 2}(t)\}. \tag{9}$$

The previous formula can be easily extended to multilevel thresholding of an image. Assuming that there are M-1 thresholds, $\{t_1, t_2, \ldots, t_{M-1}\}$, which divide the original image into M classes: C_1 for $[1, \ldots, t_1]$, C_2 for $[t_1+1, \ldots, t_2]$, … , C_i for $[t_{i-1}+1, \ldots, t_i]$, …, and C_M for $[t_{M-1}+1, \ldots, L]$, the optimal thresholds $\{t_1^*, t_2^*, \ldots, t_{M-1}^*\}$ are chosen by maximizing $\sigma_B^{\ 2}$ as follows:

$$\{t_1^*, t_2^*, \ldots, t_{M-1}^*\} = \underset{1 \le t_1 < \ldots < t_{M-1} < L}{\text{Arg} \quad \text{Max}} \ \{\sigma_B^{\ 2}(t_1, t_2, \ldots, t_{M-1})\}, \tag{10}$$

$$\text{where } \sigma_B^{\ 2} = \sum_{k=1}^{M} \omega_k (\mu_k - \mu_T)^2, \tag{11}$$

with

$$\omega_k = \sum_{i \in Ck} p_i, \tag{12}$$

$$\mu_k = \sum_{i \in Ck} i\, p_i / \omega(k). \tag{13}$$

The ω_k in Eq. (12) is regarded as the zeroth-order cumulative moment of the *kth* class C_k, and the numerator in Eq. (13) is regarded as the first-order cumulative moment of the *kth* class C_k; that is,

$$\mu(k) = \sum_{i \in Ck} i\, p_i. \tag{14}$$

3. AN ALTERNATIVE FORMULATION FOR OTSU'S METHOD

Regardless of the number of classes being considered during the thresholding process, the sum of the cumulative probability functions of M classes equals one, and the mean of the image is equal to the sum of the means of M classes weighted by their cumulative probabilities; that is

$$\sum_{k=1}^{M} \omega_k = 1, \tag{15}$$

and

$$\mu_T = \sum_{k=1}^{M} \omega_k \mu_k. \tag{16}$$

Using Eqs. (15) and (16), the between-class variance in Eq. (11) of the thresholded image can be rewritten as

$$\sigma_B^{\ 2}(t_1, t_2, \ldots, t_{M-1}) = \sum_{k=1}^{M} \omega_k \mu_k^{\ 2} - \mu_T^{\ 2}. \tag{17}$$

Because the second term in Eq. (17) is independent of the choice of the thresholds $\{t_1, t_2, \ldots, t_{M-1}\}$, the optimal thresholds $\{t_1^*, t_2^*, \ldots, t_{M-1}^*\}$ can be chosen by maximizing a modified between-class variance $(\sigma_B')^2$, defined as the summation term on the right-hand side of Eq. (17). In other words, the optimal threshold values $\{t_1^*, t_2^*, \ldots, t_{M-1}^*\}$ is chosen by

$$\{t_1^*, t_2^*, \ldots, t_{M-1}^*\} = \underset{1 \le t_1 < \ldots < t_{M-1} < L}{\text{Arg Max}} \{ (\sigma_B')^2 \{\{ t_1, t_2, \ldots, t_{M-1}\} \} \tag{18}$$

where $$(\sigma_B')^2 = \sum_{k=1}^{M} \omega_k \mu_k^{\ 2}. \tag{19}$$

According to the criteria of both Eq. (10) for $\sigma_B^{\ 2}$ and (18) for $(\sigma_B')^2$ to find the optimal thresholds, the search range for the maximal σ_B^2 and the maximal $(\sigma_B')^2$ is $1 \le t_1 < L-M+1$, $t_1+1 \le t_2 < L-M+2$, …, and $t_{M-1}+1 \le t_{M-1} < (L-1)$, as shown in Fig. 1. This exhaustive search involves $(L-M+1)^{M-1}$ possible combinations. Moreover, comparing Eq. (19) with Eq. (11), we find that the subtraction in Eq. (11) is not necessary. Thus, Eq. (19) is the better equation since it eliminates M $(L-M+1)^{M-1}$ subtractions from the threshold computations.

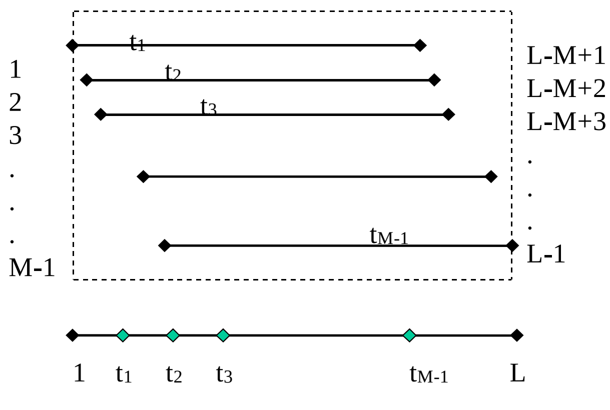


Fig. 1. Search range for $\{t_1, t_2, \ldots, t_{M-1}\}$.

4. THE FAST ALGORITHM FOR MULTILEVEL OTSU'S METHOD

As mentioned previously, finding a modified between-class variance $(\sigma_B')^2$ using Eq. (19) necessarily requires pre-computing the zeroth-order moment ω_k and the first-order moment $\mu(k)$ of the *kth* class. However, to obtain ω_K and $\mu(k)$, $1 \le k \le M$, requires much iteration due to the summation in Eqs. (12) and (14). To further reduce the computations, the u-v interval zeroth-order moment P(u, v) and the u-v interval first-order moment S(u, v) of a class with gray levels from u to v are first defined as

$$P(u,\ v) = \sum_{i=1}^{v} P_i\ , \tag{20}$$

and

$$S(u,\ v) = \sum_{i=1}^{v} i\ P_i \tag{21}$$

For index u = 1, Eqs. (20) and (21) can be rewritten recursively as

$$P(1, v+1) = P(1, v) + p_{v+1} \text{ and } P(1, 0) = 0, \tag{22}$$

$$S(1, v+1) = S(1, v) + (v+1)\ p_{v+1} \text{ and } S(1,0) = 0. \tag{23}$$

where p_{v+1} is the probability of the gray level being v+1.

From Eqs. (20)-(23), it follows that

$$P(u, v) = P(1, v) - P(1, u-1) \tag{24}$$

and

$$S(u, v) = S(1, v) - S(1, u-1). \tag{25}$$

Eq. (12), ω_k is computed as

$$\omega_k = \sum_{i \in Ck} p_i = \sum_{t_k+1}^{t_k} p_i = \sum_{1}^{t_k} p_i - \sum_{1}^{t_{k-1}} p_i$$

Combining this with Eqs. (2) and (24), ω_k can be written as

$$\omega_k = P(1, t_k) - P(1, t_{k-1}) = P(t_{k-1}+1, t_k). \tag{26}$$

Similarly, $\mu(k)$ is given by

$$\mu(k) = S(1, t_k) - S(1, t_{k-1}) = S(t_{k-1}+1, t_k). \tag{27}$$

In Eqs. (26) and (27), note that t_0 and t_M are defined as $t_0=0$ and $t_M=L$, respectively.

For all possible intensities from u to v, the u-v interval zeroth-order moment P(u, v) and u-v interval first-order moment S(u, v) of probability can be stored in look-up tables, as shown in Table 1 and Table 2. The values in the first rows of the tables are determined by using the recursive forms of P(1, v+1) and S(1, v+1) given in Eqs. (22) and (23).

Table 1. The u-v interval zeroth-order moment P(u, v) for intensities u to v.

u \ v	1	2	…	i	…	L
1	P(1,1)	P(1,2)		P(1,i)		P(1,L)
2	0	P(2,2)		P(2,i)		P(2,L)
…	0	0				
i	0	0	0	P(i, i)		P(i, L)
…	0	0	0	0		
L	0	0	0	0	0	P(L,L)

Table 2. The u-v interval first-order moment S(u, v) for intensities u to v.

u \ v	1	2	…	i	…	L
1	S(1,1)	S(1,2)		S(1,i)		S(1,L)
2	0	S(2,2)		S(2,i)		S(2,L)
…	0	0				
i	0	0	0	S(i, i)		S(i, L)
…	0	0	0	0		
L	0	0	0	0	0	S(L,L)

The values in remaining other rows are determined from Eqs. (24) and (25). We can see FROM Tables 1 and 2 that the computations of ω_K and $\mu(k)$ can be obtained directly through the use of look-up tables.

From the above description, we know that the possible thresholds $\{t_1, t_2, \ldots, t_{M-1}\}$ are the ranges $1 \le t_1 < L-M+1$, $t_1+1 \le t_2 < L-M+2$, …, and $t_{M-1}+1 \le t_{M-1} < (L-1)$. In original method by Otsu, the computations of ω_k and $\mu(k)$ are performed by Eqs. (12) and (14) for each threshold $\{t_1, t_2, \ldots, t_{M-1}\}$, and the computation complexity is bounded by O(L-M) additions. On the other hand, in our method the computations of ω_K and $\mu(k)$ are given by Eqs. (26) and (27), for the same threshold, and the computation complexity is bounded by O(M) index operations. Thus, in the process of finding the optimal multilevel threshold $\{t_1^*, t_2^*, \ldots, t_{M-1}^*\}$, the computation complexities for ω_k and $\mu(k)$ are $O((L-M)^M)$ additions Eqs. (12) and (14) and $O((L-M)^{M-1})$ index operations Eqs. (26) and (27), respectively.

By using Tables 1 and 2, the computations required for ω_k and $\mu(k)$ are significantly reduced. However, to compute the *k-th* component $(\omega_k\mu_k^2)$ of the modified between-class variance $(\sigma_B')^2$, we still need one division for computing μ_k and two multiplications for $\omega_k\mu_k^2$, as shown in Eqs. (13), (14) and (19). Thus, the total computation of $\{t_1^*, t_2^*, \ldots, t_{M-1}^*\}$ requires $M(L-M+1)^{M-1}$ divisions, $2M(L-M+1)^{M-1}$ multiplications and $(M-1)(L-M+1)^{M-1}$ additions for the summation of $\omega_k\mu_k^2$, as shown in Table 3. From Eqs. (19), (26) and (27), the modified between-class variance $(\sigma_B')^2$ can be rewritten as

$$(\sigma_B')^2(t_1, t_2, \ldots, t_i, \ldots, t_{M-1}) = H(1, t_1) + H(t_1+1, t_2) + \ldots + H(t_{M-1}+1, L), \tag{28}$$

where the modified between-class variance of class C_i is defined as

$$H(t_{i-1}+1, t_i) = S(t_{i-1}+1, t_i)^2 / P(t_{i-1}+1, t_i). \quad (29)$$

Table 3. Computations required for Eq. (18) through index operations of Table P(u, v) and Table S(u, v), and $\mu_k = \mu(k)/\omega(k)$.

$(\sigma_B')^2$	P Table	S Table	any combination				Total computation
			$\omega(k)$	$\mu(k)$	μ_k	$\sum_{k=1}^{M} \omega(k)\mu_k^2$	
Addition	L	L				M-1	$(M-1)(L-M+1)^{M-1}+2L$
Subtraction	L(L-1)/2	L(L-1)/2	0			0	L(L-1)
multiplication		L				2M	$2M(L-M+1)^{M-1}L$
Division		0			M		$M(L-M+1)^{M-1}$
direct index			M	M			$2M(L-M+1)^{M-1}$
Combinations	1		$(L-M+1)^{M-1}$				

Table 4. Computations required for Eq. (18) through index operations of Table H(u, v).

$(\sigma_B')^2$	P Table	S Table	H Table	$\sum_{k=1}^{M} H(t_{k-1}+1, t_k)$	Total computation
addition	L	L		M-1	$(M-1)(L-M+1)^{M-1}2L$
subtraction	L(L-1)/2	L(L-1)/2	0	0	L(L-1)
multiplication		L		0	L
division		0	L(L+1)	0	L(L+1)
direct index				M	$M(L-M+1)^{M-1}$
combinations	1			$(L-M+1)^{M-1}$	

With the aid of Table 1, Table 2 and Eq. (29), a look-up table H(u, v), $1 \le u \le v \le L$, can be created prior to computing the modified between-class variance $(\sigma_B')^2$. Then, using Eq. (29), for each possible threshold selection $\{t_1, t_2, \ldots, t_{M-1}\}$, the modified between-class variance $(\sigma_B')^2$ can be obtained through H(u, v) along with (M-1) additions. Thus, the large number of multiplications and divisions in Eq. (18) can also be eliminated. Table 4 lists the total computations for Eq. (18) by using H(u, v). It is from Tables 3 and 4 clear that the maximum of $(\sigma_B')^2$ can be obtained by $M(L-M+1)^{M-1}$ index operations through H(u, v) instead of $2M(L-M+1)^{M-1}$ index operations through look-up tables P(u, v) and S(u, v), $M(L-M+1)^{M-1}$ division for μ_k and $2M(L-M+1)^{M-1}$ multiplications for $\omega_k\mu_k^2$. Obviously employing look-up table H(u, v) is an efficient way to compute the modified between-class variance $(\sigma_B')^2$. Furthermore, the more classes required, the more time saved through indexing and the memory space for storing H(u, v) is half that required for storing both P(u, v) and S(u, v). The memory space required for storing H(u, v) is L(L+1)/2 units. Thus, using H(u, v) not only saves computation, but also reduces the amount of memory needed.

5. EXPERIMENTAL RESULTS

For evaluating the performance of our proposed method versus the conventional Otsu's method, four images (F16 jet, House, Lena and Peppers) are chosen; these are shown in Fig. 2. All have 256 × 256 pixels with 256 gray level intensities. Fig. 3 shows their respective histograms. To implement Otsu's method, we programmed Eqs. (10) and (11), while our proposed method was implemented using Eqs. (18) and (28). The algorithms are coded in Borland C Version 4.5 and are run on a 100 MHz Pentium II personal computer, in Microsoft Windows 95 Operating system.

Table 5. Thresholds and computation times for the test images.

Images	thresholds				computation time							
					Otsu's method with recursion without recursion				proposed method			
	2	3	4	5	2	3	4	5	2	3	4	5
F16 Jet	156	111 172	96 149 191	86 130 171 202	<1s <1s	<1s 1s	5s 70s	6m 1h	<1s	<1s	1s	37s
House	147	88 154	86 130 177	64 92 131 178	<1s <1s	<1s 1s	6s 91s	7.5m 1. 5h	<1s	<1s	1s	68s
Lena	101	77 145	56 106 159	46 83 119 164	<1s <1s	<1s 2s	9s 166s	12.0m 2.5h	<1s	<1s	1s	107s
Peppers	102	81 142	43 98 152	40 88 134 173	<1s <1s	<1s 1s	7s 105s	8.5m 1. 7h	<1s	<1s	1s	77s

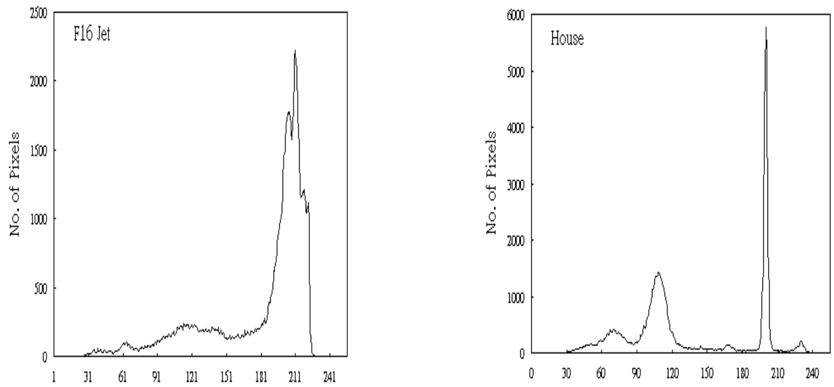
The threshold selection values and computation time for the tested images are listed in Table 5. Fig. 4 to 7 show the resulting images by the bi-level, tri-level, four-level and five-level threshold selections respectively. Because time is measured in while seconds, the true difference between our method and Otsu's method is indistinguishable for bi-level and tri-level cases. However, for the four-level selection, using the recursive forms of ω_k and $\mu(k)$ reduces the processing time from more than 70 seconds to less

(a) F-16 Jet (b) House

(c) Lena (d) Peppers

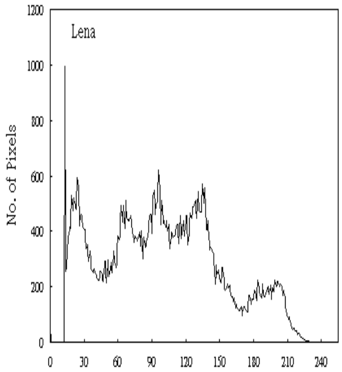
Fig. 2. Test images.

than 9 seconds. Moreover, the time can be reduced to less than one second by using look-up table H(u, v). For the five-level threshold selection, using the recursive forms of ω_k and $\mu(k)$ decreases the processing time from more than 1 hour to less than 12 minutes, and to less than 107 seconds when using H(u, v).

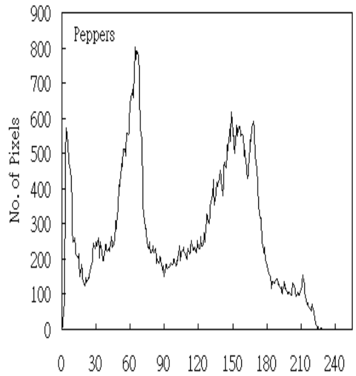


(a) (b)

Fig. 3. Histograms of test images.



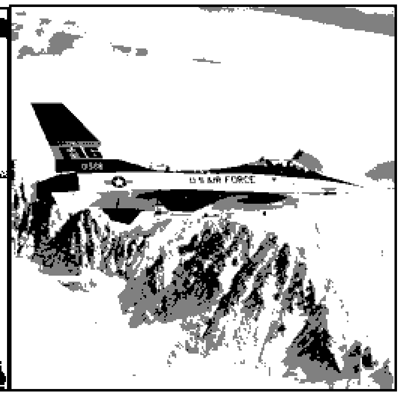
(c)



(d)

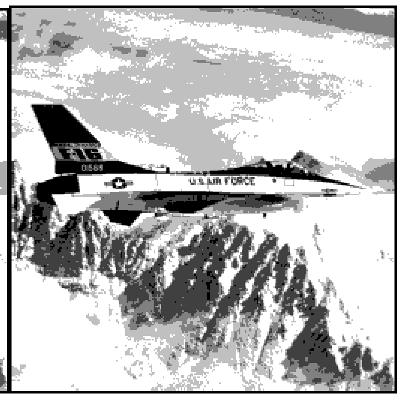
Fig. 3. (Cont'd) Histograms of test images.

(a) bi-level



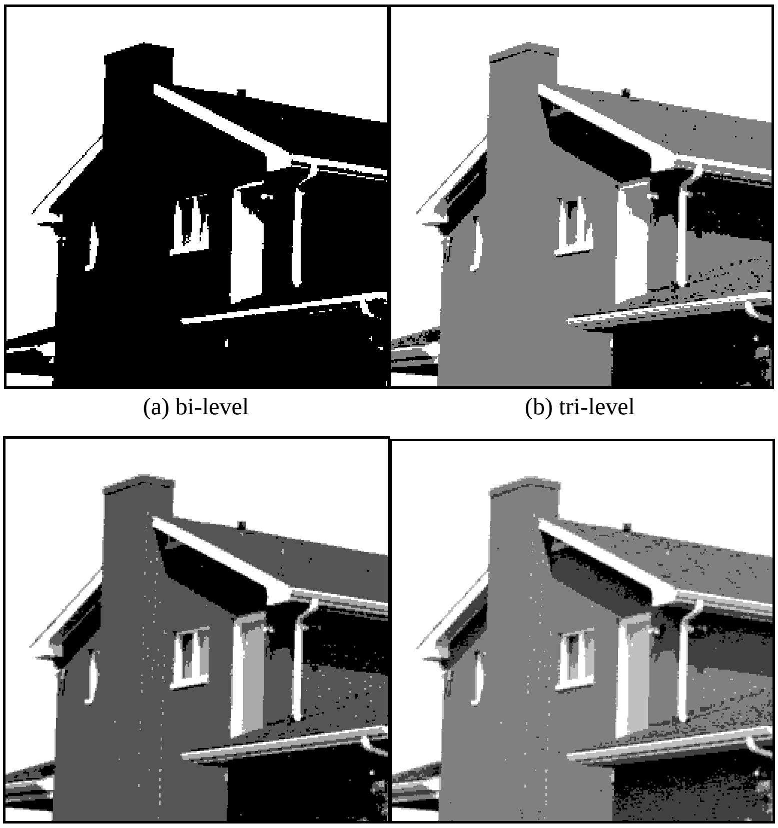
(b) tri-level

(c) four-level



(d) five-level

Fig. 4. Thresholded images for Fig. 2(a).



(a) bi-level (b) tri-level

(c) four-level (d) five-level

Fig. 5. Thresholded images for Fig. 2(b).

(a) bi-level (b) tri-level

Fig. 6. Thresholded images for Fig. 2(c).

(c) four-level (d) five-level

Fig. 6. (Cont'd) Thresholded images for Fig. 2(c).

(a) bi-level (b) tri-level

(c) four-level (d) five-level

Fig. 7. Thresholded images for Fig. 2(d).

6. CONCLUSIONS

In this paper, a fast and efficient recursive algorithm along with a look-up table has been developed for one-dimensional multilevel Otsu's thresholding. This algorithm maximizes a modified between-class variance $(\sigma_B^{'})^2$ instead of maximizing the conventional between-class variance σ_B^2 as a criterion. For M multilevel threshold selection, we showed that maximizing the modified between-class variance less computation than maximizing the conventional between-class variance. The formulas for ω_k and $\mu(k)$ for the modified between-class variance are written in recursive form, which reduces the complexity of computation for ω_k and $\mu(k)$ combined from O(L-M) additions to O(M) index operations. Moreover, determining $\omega_k\mu(k)^2$ by indexing in H(u, v) is more efficient than that by applying arithmetic operations. For five-level threshold selection, our experimental results show that the processing time of the look-up table H(u, v) is less than 107 seconds, while the of conventional Otsu's method with the recursive forms for ω_k and $\mu(k)$ takes more than six minutes, and without recursion, Otsu's method needs more than one hour. Thus, for image segmentation using thresholds derived from maximizing between-class variance, our new algorithm is a significant improvement over earlier methods.

It is also worth mentioning that our algorithm can be applied to other 1D multilevel threshold selections, such as entropic thresholding [1, 7] and correlation measure [4], whose criteria apparently have a recursive form.

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